NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science Honours in Applied Mathematics	
QUALIFICATION CODE: 08BSMH	**LEVEL:** 8
COURSE CODE: ACA801S	**COURSE NAME:** ADVANCED COMPLEX ANALYSIS
SESSION: JULY 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof F. MASSAMBA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided.   2. Show clearly all the steps used in the calculations   3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [15 marks]

1.1 Define the Cauchy Principal Value and hence [2]

Evaluate the following:

1.1.1 $P.V. \int_{-\infty}^{\infty} \frac{x \sin x}{x^2+4}$ [6]

1.1.2 $P.V. \int_{-\infty}^{\infty} \frac{1}{(x^2+4)^3}$ [7]

Problem 2 [30 marks]

2.1 Determine the order of the pole of each of the following functions at the indicated point:

2.1.1 $f(x) = \frac{1}{z \sin z}$ at $z_0 = 0$; [6]

2.1.2 $f(x) = \frac{e^{z^2}-1}{z^4}$ at $z_0 = 0$; [6]

2.2 Show that the functions given by $f(x) = \frac{\sin z}{z}$ at $z_0 = 0$ and $g(x) = \frac{e^{z-1}-1}{z-1}$ at $z_0 = 1$ possess a removable singularity at the indicated point. [9]

2.3 For the given functions $f(z) = (z^2-1)\frac{1}{z-1}$ and $g(x) = \frac{z^2}{(z-i)^3}$, determine whether they possess: [9]
(i) Removable singularity;
(ii) Pole(s), or
(iii) Essential singularity.
If it is a pole, then determine the order of the pole.

Problem 3 [25 marks]

Let $\sum_{k=0}^{\infty} a_k(z-c)^k$ be a convergent power series and $\varepsilon > 0$ such that $B_\varepsilon(c) \subset D(c, R)$, where $D(c, R)$ is the disk of convergence of the power series.
Let $f\colon B_\varepsilon(c) \to \mathbb{C}$ be defined by

$$f(z) := \sum_{k=0}^{\infty} a_k(z-c)^k.$$

3.1 Prove that f is n-times differentiable for all $n \in \mathbb{N}$ and that

$$f^{(n)}(z) = \sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k(z-c)^{k-n}$$

for all $n \in \mathbb{N}$ and all $x \in B_\varepsilon(c)$. With respect to differentiability what kind of function is f? [15]

3.2 Show that

$$\frac{f^{(n)}(c)}{n!} = a_n, \quad \text{for all } n \in \mathbb{N}_0.$$

What does this mean for the power series? [7]

3.2 What is the Taylor series of f at c? [3]

Problem 4 [30 marks]

4.1 State the Laurent series Theorem for a function of complex variable. [4]

4.2 Find the Laurent series of $f(z) = \dfrac{1}{1-z}$ for $1 < |z|$. [7]

4.3 Let $f\colon \mathbb{C} \setminus \{0\} \to \mathbb{C}$ be defined by

$$f(z) := e^{-\frac{1}{z}}.$$

4.3.1 Find the Laurent series of f about $z_0 = 0$. [5]

4.3.2 What kind of singularity is $z_0 = 0$? How does f behave in the vicinity of $z_0 = 0$? [5]

4.3.3 State the residue Theorem. [4]

4.3.4 Find [5]

$$\int_{C_1(0)} e^{-\frac{1}{\zeta}} d\zeta$$

END